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COMBINED FREE AND FORCED CONVECTION FROM AN ISOTHERMAL VERTICAL PLATE

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Abstract—The case of laminar free convection from a vertical plate with external assisting flow was investigated.

Temperature and velocity profiles were obtained using hot wire anemometry. Deduced heat-transfer rates, from the temperature profiles, were compared with the theoretical values previously presented by other investigators.

In a different experimental set up the interferometric technique was employed to obtain local heat-transfer rates, and these were compared with theory and other published data.

NOMENCLATURE

C_p,	specific heat of the fluid;
g,	acceleration due to gravity;
G_x,	local Grashof number $\rho^2\beta g(T_\omega - T_\infty)x^3/\mu^2$;
h_c,	convective coefficient;
k,	thermal conductivity of the fluid;
N_x,	local Nusselt number $h_c x/k$;
P,	Prandtl number $C_p\mu/k$;
R_x,	local Reynolds number $\rho ux/\mu$;
T,	temperature;
u,	velocity;
x,	distance along the length of the plate;
y,	distance perpendicular to the plate.

Greek symbols

β,	coefficient of thermal expansion of the fluid;
μ,	absolute viscosity of the fluid;
ν,	kinematic viscosity of the fluid;
ρ,	density of the fluid.

Subscripts

ω,	wall conditions;
∞,	free stream conditions.

INTRODUCTION

VARIOUS authors have reported on theoretical work in the field of heat transfer from a vertical heated plate with a combination of free convection and assisted external flow. Early attempts to obtain a theoretical expression for heat transfer from such a vertical plate were concerned with solutions obtained by expanding the velocity and temperature functions into a series form of the parameter G_x/R_x^2. Such work (e.g. Szewczyk [1] and Eshghy [2]) was done on the basis of extending the range of forced convection theory. Merkin [3] has obtained a complete solution for the case of Prandtl number = 1·0 by a combination of series expansion and numerical integration and showed that the earlier attempts to obtain a solution by series expansion were in error. Attempts have also been made to analyze the problem using integral equation methods (e.g. Oosthuizen [4]), but it can be shown that the accuracy of these analyses is in doubt.

Recently Lloyd and Sparrow [5] have obtained a complete solution to the combined free and forced flow regime for a constant temperature plate by using the local similarity method, while Oosthuizen and Hart [6] have presented solutions to both the problem of constant heat flux and constant wall temperature by solving the equations by a finite difference method. Wilks [7] has presented work on the constant heat-flux which supplements the work by Lloyd and Sparrow [5].

The only experimental works in the field appear to be those of Kliegel [8] who used the interferometric method to determine local heat fluxes from a plate, and of Hall and Price [9] who investigated and measured the effect of superimposing a forced upward flow on a turbulent ($G_x > 10^8$) free convection boundary layer. No work has been reported on the determination of local temperature and velocity profiles as well as heat-transfer rates, for the flow regime of laminar natural convection with assisting external flow, from an isothermal vertical plate.

EXPERIMENTAL METHOD

The first of the two experimental set ups employed in this work (described in the appendix) was designed to provide temperature and velocity profiles in the vicinity of an isothermal vertical plate in free convection with assisting external flow.

Temperature and velocity determinations were made by a traversing hot-wire anemometer. The hot wire probe was calibrated for low velocities in a varying temperature field before using it to probe the boundary layer on the plate. (See Appendix.)

Preliminary tests with the apparatus were carried out in the purely free convection regime to determine the accuracy of the measuring technique and verify the

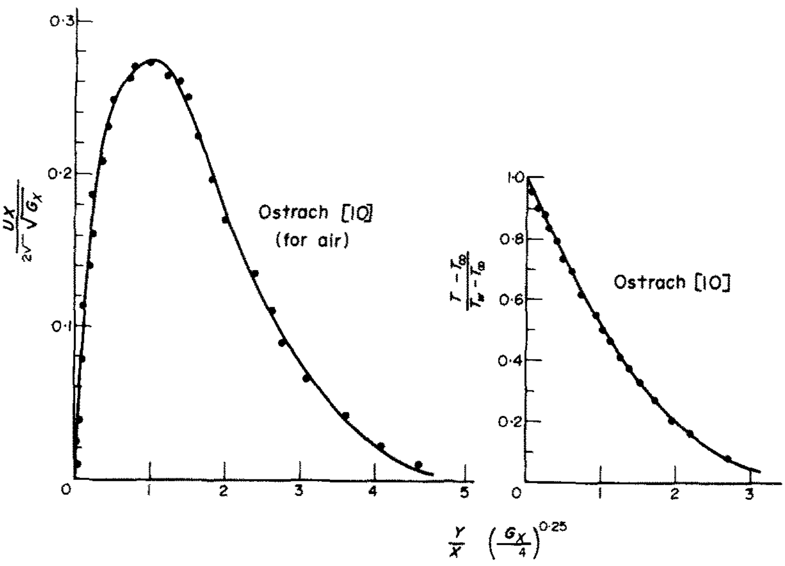

FIG. 1. Velocity and temperature profiles from a vertical plate in free convection.

calibration of the probe. The results of these preliminary tests are presented in Fig. 1, as temperature and velocity profiles from a vertical heated plate in free convection, and comparison is made with the theory presented by Ostrach [10]. In view of the good agreement observed, the author proceeded in the combined mode phase of the experiments.

Tests, in the free convection with assisting external flow regime, were carried out for the range of the governing dimensionless parameter G_x/R_x^2 from 0·40 to 9·0. This was effected by probing the boundary layer in the vicinity of the heated plate, at various free stream velocities, and at different x's. Care was taken to probe the boundary layer well away from the leading edge of the plate, in order to avoid its effects, in view of the findings reported in [11].

The temperature and velocity profiles using the above mentioned technique appear in Fig. 2.

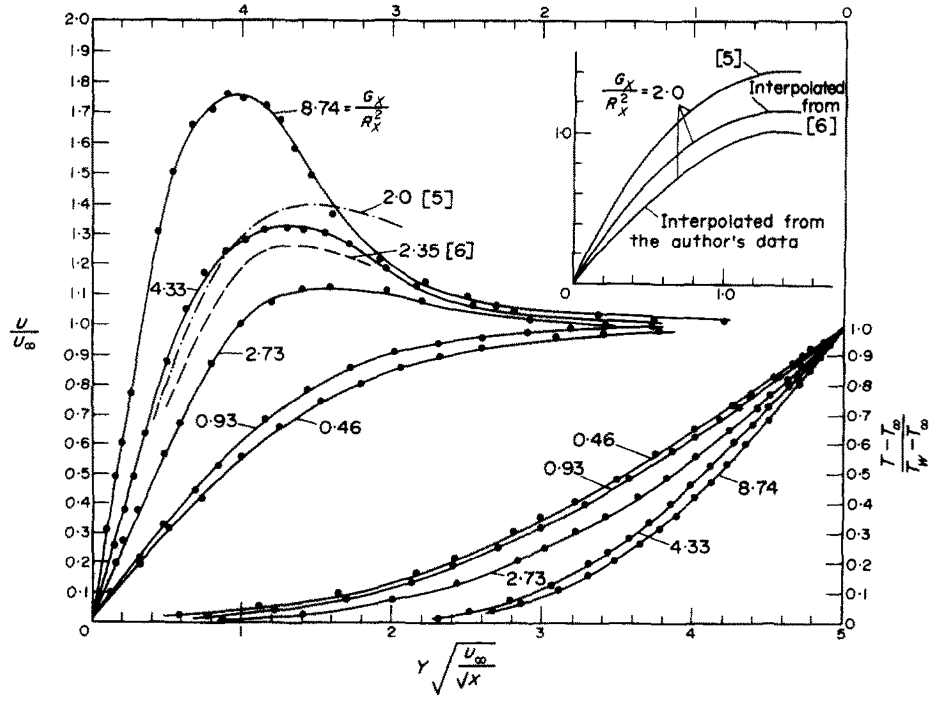

FIG. 2. Temperature and velocity profiles from a vertical plate in free convection with assisting external flow.

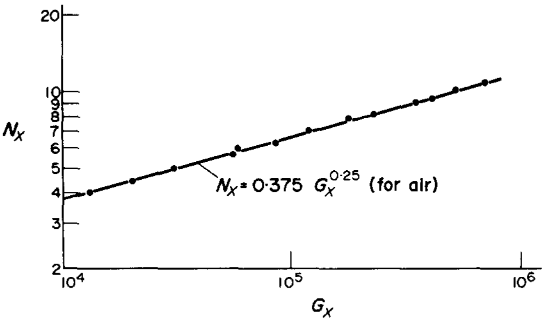

FIG. 3. Free convection heat-transfer rates from a vertical plate in air.

The second experimental apparatus (see Appendix) was designed to provide local heat-transfer rates by an independent method. Using the interferometric set up described in the Appendix, the accuracy of the method was tested in a preliminary test in the purely free convection regime. The results of this test appear in Fig. 3 and compare favorably with those reported in [11]. Subsequently, a number of interferograms were obtained at various plate temperatures with a range of free stream velocities (1–20 cm/s) and different x's for the combined flow, and the results are reported in Fig. 5.

COMPARISON OF THEORETICAL AND EXPERIMENTAL VALUES

A comparison of theory and the experimental work reported here is made considering the temperature and velocity profiles, and heat-transfer rates.

Considering the first experimental technique (hot wire anemometry), it is seen that care was taken to ensure the accuracy of the method by duplicating data for a heated vertical plate in air, in free convection. The results appear in Fig. 1 where the experimental points are compared with the theory presented by Ostrach [10].

The basic difficulty in probing for the velocity profile from a heated surface is that the probe operates in a temperature gradient as well; therefore, a calibration is necessary. The author believes that the calibrations were adequate in view of the agreement between experimental results and theory shown in Fig. 1.

The author's experimental temperature profiles for the combined mode are shown in Fig. 2. These results, when compared with the ones reported in [5] and [6], show good agreement. For the sake of clarity the temperature profiles of [5] and [6] were not indicated in Fig. 2.

The correlation between theory and experiment is, however, not so good for velocity profiles.

The author's experimental velocity profiles appear in Fig. 2 together with a sample of those reported in [5] and [6]. Interpolation of the author's results, to obtain a velocity profile for the value of $G_x/R_x^2 = 2{\cdot}0$, is shown in the upper right corner of Fig. 2. Included in this figure are the velocity values of Lloyd and Sparrow [5] at $G_x/R_x^2 = 2{\cdot}0$ and the values of Oosthuizen and Hart [6] interpolated from their theoretical values. Note that this was done solely because of the relatively small amount of data reported in [5].

The results of Oosthuizen and Hart [6] are approximately 25 per cent and Lloyd and Sparrow's [5] results approximately 70 per cent higher than the author's experimental values.

It is obvious that having the temperature profiles for the combined mode, the local Nusselt numbers may be obtained from the fluid temperature gradient near the wall using the correlation equation

$$N_x/\sqrt{R_x} = -\frac{\partial T}{\partial\left[y\sqrt{\left(\frac{u}{\nu x}\right)}\right]}\Bigg|_{y=0} \tag{1}$$

Using the above equation, local Nusselt numbers were deduced from the author's experimental temperature profiles. The results appear in Fig. 4, and good correlation with the theory of Oosthuizen and Hart [6] is noted.

Considering the second experimental technique (interferometry), which was employed in order to obtain local heat-transfer rates by independent means as well as greater volume of data, it is seen that care was taken to ensure the accuracy of the method. Local heat-transfer rates were obtained for the purely free convective case, and the results are compared with previously published data. Figure 3 represents the local Nusselt values obtained, and comparison is made with the accepted power law relationship

$$N_x = 0{\cdot}375\,G_x^{0{\cdot}25} \tag{2}$$

reported in [11].

Since a great number of experimental values were needed for the range of $0{\cdot}1 < G_x/R_x^2 < 1000$, the hot wire anemometry being cumbersome was abandoned and interferometry was employed.

The results of the interferometric method are reported in Fig. 5, and comparison is made with the theory presented by Lloyd and Sparrow [5] and Oosthuizen and Hart [6].

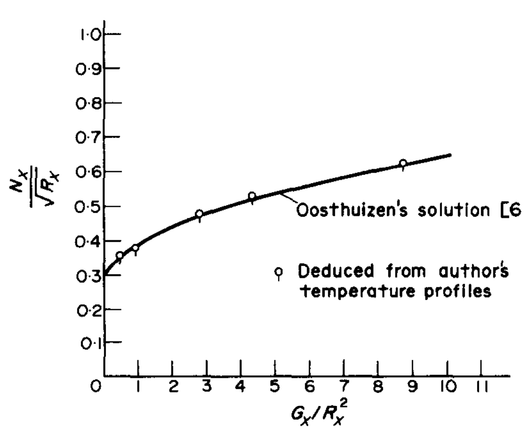

FIG. 4. Heat-transfer rates in combined free and forced (assisting) convection from a vertical plate.

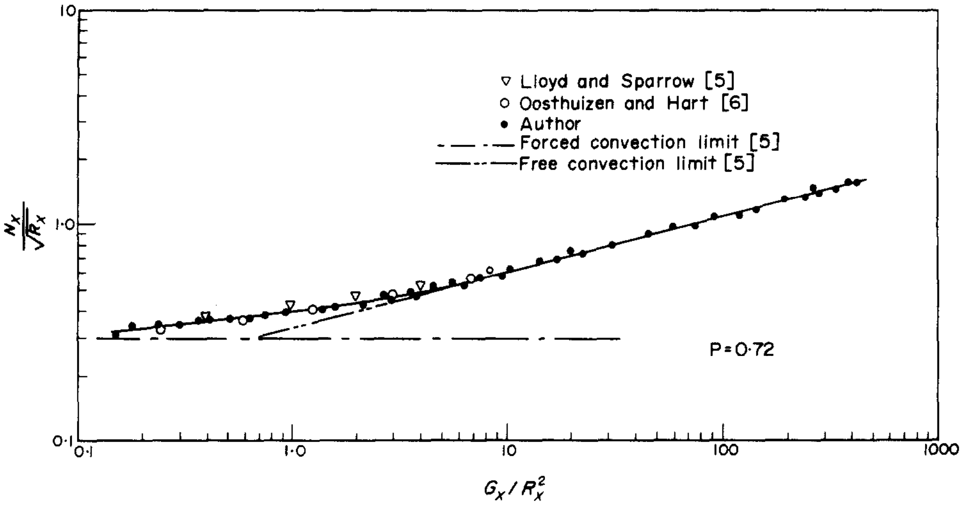

FIG. 5. Local Nusselt number from a vertical isothermal plate in free convection with assisting external flow.

CONCLUSIONS

In spite of poor agreement between theoretical and experimental velocity profiles, the experimental temperature profiles correlate well; and the resulting local Nusselt numbers can be obtained accurately. This can only hold if the solution is relatively insensitive to the velocity profile because a comparison between the theoretical values of Lloyd and Sparrow [5] and Oosthuizen and Hart [6] shows that good velocity profile correlation is not important for a good Nusselt number correlation!

The discrepancy between the author's velocity profiles and those of Oosthuizen and Hart [6] may be due to the latter's assumption that at the leading edge there is no normal component of velocity. It has been shown in [11], however, that there is in fact a normal velocity component, and the flow starts ahead of the leading edge; therefore, the profiles would be affected. Oosthuizen and Hart [6] have suggested an alternative method of determining suitable initial conditions by applying a series solution applicable to small values of x to give conditions near the leading edge. This method was not used, but it would be of interest to apply such a method to determine whether better agreement with the experimental values could be obtained. It is suggested here that the leading edge does not affect the shape or form of the velocity profiles along the length of the plate, but rather their relative position from the leading edge.

As far as the temperature profiles, it appears that Oosthuizen and Hart's [6] assumption that near the wall the temperature profile can be approximated by a third degree polynomial is valid. This is borne out by the fact that their solution for local Nusselt numbers is in good agreement with the author's results deduced from the experimental temperature profiles, as it is shown in Fig. 4.

It is a fact that the ratio of G_x/R_x^2 gives a qualitative indication of the influence of forced flow in a natural convection process. This influence becomes significant when the square of the Reynolds number is of the same order of magnitude as the Grashof number. Furthermore, it is expected that at any value of G_x/R_x^2 between the limits of purely forced and purely free convection flows, the Nusselt number is higher than it would be in either of these modes alone.

In Fig. 5 the author's experimental data are compared with the solutions presented in [5] and [6]. The free convection and forced convection limits are indicated, and it is seen that the free convection limit is approached at approximately the value of $G_x/R_x^2 = 3{\cdot}0$. On the other end, the forced convection limit is approached at approximately $G_x/R_x^2 = 0{\cdot}10$; so it can be concluded that the external flow aids and increases the heat transfer in the region of $0{\cdot}1 < G_x/R_x^2 < 3{\cdot}0$.

Kliegel's [8] data were not available to the author except in the form reported in [5] and [6], and aside from experimental scatter, they appear to be essentially the same as those presented in this work.

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REFERENCES

1. A. A. Szewczyk, Combined free and forced convection flow and heat transfer, *J. Heat Transfer* **86**, 501 (1964).
2. S. Eshghy, Forced flow effects on free convection flow and heat transfer, *J. Heat Transfer* **86**, 290 (1964).
3. J. H. Merkin, The effect of buoyancy forces on the boundary layer flow over a semi-finite vertical flat plate in a uniform free stream, *J. Fluid Mech.* **35**, 439 (1969).
4. P. H. Oosthuizen, A note on the combined free and forced convective laminar flow over a vertical isothermal plate, *S. Afr. Mech. Engr* **15**, 8 (1965).

5. J. R. Lloyd and E. M. Sparrow, Combined forced and free convection flow on vertical surfaces, *Int. J. Heat Mass Transfer* **13**, 434 (1970).
6. P. H. Oosthuizen and R. Hart, A numerical study of laminar combined convective flow over flat plates, Rep. 3/71, Queen's University (1971).
7. G. Wilks, Combined forced and free convection flow on vertical surfaces, *Int. J. Heat Mass Transfer* **16**, 1958 (1973).
8. J. R. Kliegel, Laminar free and forced convection heat transfer from a vertical flat plate, Ph.D. Thesis University of California (1959). (Data appeared in References 5 and 6.)
9. W. B. Hall and P. H. Price, Mixed forced and free convection from a vertical heated plate to air, in *Heat Transfer*, Vol. IV NC3.3, Fourth International Heat Transfer Conference, Paris-Versailles (1970).
10. S. Ostrach, An analysis of laminar natural convection flow and heat transfer about a flat plate parallel to the direction of the generating body force, NACA TN 2635 (1952).
11. J. Gryzagoridis, Leading edge effects on the Nusselt number for a vertical plate in free convection, *Int. J. Heat Mass Transfer* **16**, 517 (1973).
12. P. Almquist and E. Legath, The hot wire anemometer at low air velocities, DISA 2,3 (1965).
13. M. A. Connor, Velocity, temperature and turbulence measurements in air under combined free and forced convection conditions, Ph.D. Thesis University of Cape Town (1971).

APPENDIX

(a) *Hot Wire Anemometry*

A vertical plate was constructed having the following characteristics: length 300 mm, width 180 mm and thickness 6 mm. A paxolin former was used, wound with resistance tape and sandwiched between two 16BSW chromed steel plates, separated by thin mica sheets. The resistance tape used was nichrome ribbon 1·4 mm wide with a resistance of 6·6 Ω/m and was wound laterally round the former. After every eleven turns, a connection point was soldered to the ribbon to enable the amount of heat dissipated in each set of windings to be controlled externally. Wall temperature measurements were made with seven iron constantan thermocouples embedded along the length of the steel plates.

The temperature of the plate was controlled by varying the current into each resistance bank wound on the paxolin former by placing an external variable resistance in parallel with each bank.

The plate was suspended in the "wind tunnel" by knife edges. The wind tunnel enabled the application of a uniform free stream velocity. Its test section was 230 mm square and 500 mm long, with a suitable bell mouth entrance and flow straightners. One side of the wind tunnel was constructed with a suitable slot in its center to enable the probe traversing gear to travel along the length of the plate. The flow was induced by an axial flow fan, and preliminary tests showed no variation of the free stream velocity.

Under normal circumstances measurements of flow velocity, using the hot wire technique, are very accurate. Of the three modes that a hot wire probe can be used, the constant current mode is preferred for measuring extremely low velocities because of its accuracy and resolution.

The usual procedure is to balance the constant current bridge at zero velocity and ambient temperature and then to use as the measure of velocity the voltage unbalance which appears when the wire is cooled by the fluid stream.

When probing the velocity field in the vicinity of a heated vertical plate, considerable error can result because the probe is exposed to a temperature gradient as well. When the ambient temperature changes, the hot wire's resistance changes also in the same direction so it is necessary to separate the flow and temperature effects.

The velocity calibrations of the probe were quite elaborate and lengthy and will not be discussed in great detail. It will suffice to mention that the temperature calibration was effected in an environmental test chamber capable of control stability of 0·28°C within the range of -100 to 450°C.

The probe was suitably shielded from radiation and possible convection currents in the chamber, and values of "no flow" resistance at different temperatures were obtained.

The velocity calibration at room temperature was made using the apparatus suggested by Almquist and Legath [12]. The velocity calibration at elevated temperatures was made using an apparatus constructed for the purpose by Connor [13]. This apparatus is basically a low speed wind tunnel with a heated test section. Data were collected for a number of temperatures and velocities within the range that the probe was expected to operate.

Having already the temperature "no flow" calibration, it would be easy to separate the flow and temperature effects provided the true temperature was known in the plate's boundary layer.

The temperature profile in the boundary layer was obtained using the DISA 55F05 temperature sensor which is very similar to the velocity probe. The flow sensitivity of this sensor, when operated in the constant current mode, is kept small by using very low current, 1 mA or less. At a velocity of 50 m/s the error in the temperature signal due to the velocity corresponds to about 1°C. Since very much lower velocities than 50 m/s were expected, the temperature profile was determined very accurately.

Of course, the real test whether the probes were calibrated adequately came when data were collected in the case of free convection. The results appear in Fig. 1 where very good correlation is noted with the theory.

(b) *Interferometry*

An electrically heated plate with the dimensions 6·35 cm height, 10·15 cm width, 0·5 cm thick was constructed in the same manner as described under a, above.

A suitable "wind tunnel" was constructed and fitted in the test section of the Mach–Zehnder interferometer. A small axial flow fan was used to induce flow through the test section. The flow in the wind tunnel was checked for uniformity using the hot wire probe, which was calibrated as suggested by [12].

CONVECTION LIBRE ET CONVECTION FORCEE COMBINEES SUR UNE PLAQUE VERTICALE ISOTHERME

Résumé—Le cas de la convection libre laminaire sur une plaque verticale, doublée d'un écoulement externe est étudié.

Les profils de température et de vitesse ont été obtenus à l'aide de l'anémomètre à fil chaud. Les taux de transfert de chaleur, déduits des profils de température, ont été comparés aux valeurs théoriques données antérieurement par d'autres auteurs.

La technique interférométrique a été employée dans un montage expérimental différent afin d'obtenir les taux de transfert de chaleur locaux, ces derniers ont été comparés à la théorie et à d'autres données publiées.

ÜBERLAGERTE FREIE UND ERZWUNGENE KONVEKTION VON EINER ISOTHERMEN SENKRECHTEN PLATTE

Zusammenfassung—Der Fall von laminarer freier Konvektion von einer senkrechten Platte mit äußerer Stützströmung wurde untersucht.

Temperatur- und Geschwindigkeitsprofile wurden mit Hitzdrahtanemometern bestimmt. Die aus den Temperaturprofilen abgeleiteten Wärmeübergangszahlen wurden mit theoretischen, früher von anderen Forschern angegebenen Werten verglichen. In einer weiteren Untersuchungsreihe wurden interferometrische Techniken eingesetzt, um die örtlichen Wärmeübergangszahlen zu bestimmen; diese wurden mit der Theorie und anderen veröffentlichen Daten verglichen.

СОВМЕСТНАЯ СВОБОДНАЯ И ВЫНУЖДЕННАЯ КОНВЕКЦИЯ С ИЗОТЕРМИЧЕСКОЙ ВЕРТИКАЛЬНОЙ ПЛАСТИНЫ

Аннотация — Исследовалась ламинарная свободная конвекция от вертикальной пластины к внешнему потоку. С помощью термоанемометра получены профили температуры и скорости. Коэффициенты теплообмена, рассчитанные из температурных профилей, сравнивались с теоретическими значениями, ранее представленными другими исследователями. Кроме того, на другой экспериментальной установке для получения локальных значений коэффициентов теплообмена использовалась интерферометрическая техника. Полученные результаты сравнивались с теоретическими данными и данными других исследователей.